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Relative Price Effects on Decompositions of Change in Aggregate Labor Productivity

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This paper shows that the decomposition of log-change in aggregate labor productivity (ALP) devised by Balk (2013) based on Sato-Vartia indexes is inexact when applied to GDP in chained or in constant prices so that sectoral contributions do not necessarily add up to "actual" log-change in ALP. However, this paper adjusts Balk's decomposition by incorporating "relative prices"–from the "generalized exactly additive" (GEAD) decomposition of "arithmetic change" in ALP (Dumagan, 2013)–and shows that the adjusted Balk decomposition is exact for GDP in chained or in constant prices like GEAD. An important finding is that relative prices could *reverse* the signs of sectoral contributions from Balk's inexact decomposition. Hence, results from related decompositions of log-change in ALP, e.g., those based on the Törnqvist framework, that do not explicitly recognize relative prices could be misleading and, therefore, may need reconsideration.

KEY WORDS: Relative prices; productivity change decomposition; index number theory

JEL classification: C43, O47

1. Introduction

It is economic doctrine that relative prices help in guiding resource allocation. Thus, it appears surprising that in determining intersectoral reallocation effects in decompositions of change in aggregate labor productivity (ALP), most decompositions do not *explicitly* account for the effects of differences or changes in sectoral relative prices.[1] The literature is replete with such decompositions, exemplified by the common Törnqvist index framework for log-change in ALP (e.g., Nordhaus, 2002; Stiroh, 2002; Triplett and Bosworth, 2004; Bosworth and Triplett, 2007; Jorgenson, Ho, Samuels, and Stiroh, 2007; Timmer, et.al., 2010; and Karagiannis, 2011). Similar Sato-Vartia index framework for log-change in ALP without explicit roles of relative prices are rarer, e.g., Reinsdorf and Yuskavage (2010) and Balk (2013).

Balk's (2013) decomposition is an excellent basis for comparing results *without* and *with* relative prices because, as this paper shows, it becomes exact for GDP in chained or in constant prices after incorporation of relative prices defined by the "generalized exactly additive"

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[1] Some exceptions are Tang and Wang (2004), Diewert (2010), and Dumagan (2013).

(GEAD) decomposition.[2] Moreover, Balk's framework translates to existing Törnqvist decompositions of log-change in ALP–based *directly* on sectoral value added or *indirectly* based on sectoral gross output net of intermediate inputs–by replacing the Sato-Vartia weights in Balk's framework with the corresponding Törnqvist weights. Balk himself pointed out Törnqvist results in earlier studies noted above corresponding to his results. Thus, the findings in this paper–by way of their relationship to Balk's framework–have analytic implications on Törnqvist log-change that appears to be the framework of choice in ALP change decompositions in the literature.

This paper is organized as follows. Section 2 shows that the decomposition of log-change in ALP by Balk (2013)–in its present form–is *inexact* for GDP in chained or in constant prices. That is, the sum of sectoral contributions does not necessarily equal "actual" log-change in ALP.

Section 3 incorporates into Balk's procedure "relative price"–ratio of a sectoral GDP deflator to the aggregate GDP deflator–that is the key to exactness of GEAD. With relative prices, Balk's procedure becomes an exact decomposition for GDP in chained or in constant prices like GEAD.

Henceforth, for brevity of exposition, Balk's original version that is *inexact* without relative prices will be referred to as BMB, the initials of Bert M. Balk. In contrast, this paper's "generalized exactly additive" reformulation of BMB will be referred to as GEA-BMB. From above, GEA-BMB and GEAD are both *exact* and the only difference is that GEA-BMB decomposes "log-change" while GEAD decomposes "arithmetic change" in ALP.

Section 4 presents results from applications of BMB, GEA-BMB, and GEAD. The first is an application to US Manufacturing for years 2004 and 2005 that were selected to show that GEA-BMB could yield sectoral contributions to ALP change with signs *opposite* to those by BMB. Moreover, in this case where US Manufacturing data are in chained prices based on Fisher indexes, the results show that BMB is inexact while GEA-BMB and GEAD are exact decompositions of ALP change.

It is also shown that BMB is inexact while GEA-BMB and GEAD are exact in two more cases of (a) GDP in chained prices (but based on *chained* Paasche price and Laspeyres quantity indexes) and (b) GDP in constant prices (based on *direct* Paasche price and Laspeyres quantity

[2] Dumagan (2013) coined the acronym "GEAD" to generalize the applicability to any *real* GDP, i.e., in chained or in constant prices, of the exact decomposition by Tang and Wang (2004) of ALP change in Canada and US where GDP are in chained prices based on the Fisher index.

indexes). To illustrate the above results, this paper found it convenient to use the same data in Dumagan (2013) for the Agriculture sector in Italy to represent case (a) and also for the Agriculture sector in Thailand to represent case (b).

The above illustrations for the US, Italy, and Thailand exhaust existing measures of real GDP world-wide and, therefore, suffice to establish the general applicability of this paper's analytic framework.

Section 5 concludes this paper with a summary of salient findings with their implications on earlier ALP change studies.

2. Decomposition of Log-Change in ALP

The following presentation adopts the notation in Balk (2013) for ease of comparison with this paper. In Balk's framework, the change is from period 0 to 1. The aggregate (i.e., the economy) is denoted by upper case K and a sector by lower case k. VA is nominal value added and L is labor. However, for simplicity in this paper, Balk's notation VA for value added is shortened to V and will be referred to as GDP. Moreover, GDP means "aggregate GDP" unless otherwise qualified, for example, by "sectoral GDP."

Nominal GDP and labor are additive, i.e.,

$$V^{Kt} = \sum_{k \in K} V^{kt} \quad ; \quad L^{Kt} = \sum_{k \in K} L^{kt} \quad ; \quad t = (0,1)\,. \tag{1}$$

Balk employs Sato-Vartia (Sato, 1976; Vartia, 1976) index formulas to obtain from (1) the nominal GDP and total labor indexes given by,[3]

$$\frac{V^{K1}}{V^{K0}} = \exp\left\{\sum_{k \in K} \Psi^k \ln\left(\frac{V^{k1}}{V^{k0}}\right)\right\} \quad ; \quad \frac{L^{K1}}{L^{K0}} = \exp\left\{\sum_{k \in K} \xi_L^k \ln\left(\frac{L^{k1}}{L^{k0}}\right)\right\}. \tag{2}$$

The weights are normalized logarithmic means of nominal GDP shares and of labor shares,[4]

$$\Psi^k \equiv \frac{L\left(\frac{V^{k1}}{V^{K1}}, \frac{V^{k0}}{V^{K0}}\right)}{\sum_{k \in K} L\left(\frac{V^{k1}}{V^{K1}}, \frac{V^{k0}}{V^{K0}}\right)} \quad ; \quad \xi_L^k \equiv \frac{L\left(\frac{L^{k1}}{L^{K1}}, \frac{L^{k0}}{L^{K0}}\right)}{\sum_{k \in K} L\left(\frac{L^{k1}}{L^{K1}}, \frac{L^{k0}}{L^{K0}}\right)} \quad ; \quad \sum_{k \in K} \Psi^k = \sum_{k \in K} \xi_L^k = 1\,. \tag{3}$$

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[3] While Balk (2013) did not mention Sato (1976) and Vartia (1976), (2) and (3) are Sato-Vartia indexes as they are known in the literature.

[4] By definition, the logarithmic mean of any two strictly positive numbers a and b where $a \neq b$ is $L(a,b) \equiv (a-b)/\ln(a/b)$ and $L(a,a) \equiv a$.

Sectoral GDP price index $P_V^k(1,0)$ and quantity index $Q_V^k(1,0)$ satisfy "factor reversal" by construction of Sato-Vartia indexes. That is, as shown by Balk in Eq. (12),

$$\frac{V^{k1}}{V^{k0}} = P_V^k(1,0)Q_V^k(1,0)\,. \tag{4}$$

Since factor reversal also holds at the aggregate level, (2) and (4) yield Balk's Eq. (21),

$$\ln\left(\frac{V^{K1}/L^{K1}}{V^{K0}/L^{K0}}\right) = \sum_{k\in K}\Psi^k \ln P_V^k(1,0) + \sum_{k\in K}\Psi^k \ln Q_V^k(1,0) - \sum_{k\in K}\xi_L^k \ln\left(\frac{L^{k1}}{L^{k0}}\right). \tag{5}$$

In (5), let V_r^{Kt} and V_r^{kt} be real GDP of the economy and of a sector. That is,

$$\ln\left(\frac{V_r^{K1}}{V_r^{K0}}\right) = \ln\left(\frac{V^{K1}}{V^{K0}}\right) - \sum_{k\in K}\Psi^k \ln P_V^k(1,0) \quad ; \quad \ln\left(\frac{V_r^{k1}}{V_r^{k0}}\right) = \ln Q_V^k(1,0)\,. \tag{6}$$

Therefore, (5) becomes,

$$\ln\left(\frac{V_r^{K1}/L^{K1}}{V_r^{K0}/L^{K0}}\right) = \sum_{k\in K}\Psi^k \ln\left(\frac{V_r^{k1}}{V_r^{k0}}\right) - \sum_{k\in K}\xi_L^k \ln\left(\frac{L^{k1}}{L^{k0}}\right). \tag{7}$$

The left-hand side of (7) is the log-change in ALP and the right-hand side may now be expressed in terms of log-change in sectoral labor productivities by,

$$\ln\left(\frac{V_r^{K1}/L^{K1}}{V_r^{K0}/L^{K0}}\right) = \sum_{k\in K}\Psi^k \ln\left(\frac{V_r^{k1}/L^{k1}}{V_r^{k0}/L^{k0}}\right) + \sum_{k\in K}\left(\Psi^k - \xi_L^k\right) \ln\left(\frac{L^{k1}}{L^{k0}}\right). \tag{8}$$

Note that total labor L^{Kt} may be treated as a *scalar*. Therefore, (3) yields,

$$\sum_{k\in K}\left(\Psi^k - \xi_L^k\right) = 0 \quad ; \quad \sum_{k\in K}\left(\Psi^k - \xi_L^k\right) \ln\left(\frac{L^{K1}}{L^{K0}}\right) = 0\,. \tag{9}$$

Combine (9) with (8) and use the definition of the total labor index in (2) to simplify and obtain,

$$\ln\left(\frac{V_r^{K1}/L^{K1}}{V_r^{K0}/L^{K0}}\right) = \sum_{k\in K}\Psi^k \ln\left(\frac{V_r^{k1}/L^{k1}}{V_r^{k0}/L^{k0}}\right) + \sum_{k\in K}\Psi^k \ln\left(\frac{L^{k1}/L^{K1}}{L^{k0}/L^{K0}}\right). \tag{10}$$

It may be recognized that (10) corresponds to Eq. (24) combined with Eq. (25) in Balk (2013).

Suppose now that US GDP in chained prices based on the Fisher index is substituted into (10). Denoting the above GDP by V_U^{Kt} and V_U^{kt} and substituting these into (10) will yield an *inexact* decomposition–referred to as BMB in the introduction–given by,

$$\ln\left(\frac{V_U^{K1}/L^{K1}}{V_U^{K0}/L^{K0}}\right) \approx \sum_{k\in K}\Psi^k \ln\left(\frac{V_U^{k1}/L^{k1}}{V_U^{k0}/L^{k0}}\right) + \sum_{k\in K}\Psi^k \ln\left(\frac{L^{k1}/L^{K1}}{L^{k0}/L^{K0}}\right). \tag{11}$$

The reason for the inexactness of BMB in (11) is that real GDPs in (10) are obtained by Sato-Vartia price indexes while those in (11) are obtained by Fisher price indexes but these indexes only *approximate* each other.[5] Moreover, (11) will hold if aggregate and sectoral GDP in constant prices are substituted in (10) because Sato-Vartia price indexes are different from *direct* Paasche price indexes that yield GDP in constant prices.

It should be emphasized that the problem with (11) is not the size of the approximation discrepancy because this discrepancy is negligible especially for US GDP (see for instance, Dumagan, 2002; and Dumagan, 2009). The problem is with the reallocation term in (11) because it depends only on changes in labor shares. "Relative prices" that on principle have a role in resource allocation are absent. The solution is to incorporate relative prices so that changes in relative prices together with changes in labor shares determine reallocation effects. It turns out that this solution has the added benefit of making (11) exact. However, the incorporation of relative prices may have dramatic results because it could reverse the signs of intersectoral reallocation effects in the second term of (11). Therefore, given the same within-sector productivity effects in the first term, the ALP contribution of a sector–the sum of the same productivity effect and the new reallocation effect–could be significantly different than originally calculated. This is shown analytically in the next section and illustrated empirically in the section after the next.

3. Incorporating Relative Prices in Log-Change ALP Decomposition

As noted above, BMB reallocation effects in (11) involve *only* changes in labor shares. In contrast, the reallocation effects in GEAD involve changes in labor shares and in relative prices.

In GEAD, relative prices are ratios of implicit sectoral GDP deflators to the GDP deflator from the national income accounts. Following the notation for US nominal GDP and GDP in chained prices in the preceding section, these implicit deflators are,

$$P_V^{Kt} = \frac{V^{Kt}}{V_U^{Kt}} \quad ; \quad P_V^{kt} = \frac{V^{kt}}{V_U^{kt}} \quad ; \quad r_V^{kt} \equiv \frac{P_V^{kt}}{P_V^{Kt}} . \tag{12}$$

"Relative price" is r_V^{kt} defined by the ratio of the sectoral GDP deflator P_V^{kt} to the GDP deflator P_V^{Kt}. It follows from (1) and (12) that,

[5] Fisher price and quantity indexes also satisfy factor reversal (Fisher, 1922) and, thus, satisfy (5) when substituted for the Sato-Vartia indexes. This does not, however, imply that real GDP obtained by Fisher indexes may be substituted for real GDP obtained by Sato-Vartia indexes in (10) and maintain the equality. This substitution will yield (11).

$$V_U^{Kt} = \frac{V^{Kt}}{P_V^{Kt}} = \sum_{k \in K} \frac{V^{kt}}{P_V^{Kt}} = \sum_{k \in K} \left(\frac{P_V^{kt}}{P_V^{Kt}}\right) V_U^{kt} = \sum_{k \in K} r_V^{kt} \, V_U^{kt} \,. \tag{13}$$

It is important to note that (13) is true for *any* real GDP and is the basis of GEAD. Thus, GEAD applies regardless of the price index formulas underlying the implicit deflators in (12). However, the *real* valuation of $r_V^{kt} V_U^{kt}$ depends on the denominator of r_V^{kt}. It is in chained prices if the denominator is a *chained* price index but in constant prices if it is a *direct* Paasche price index.

To elaborate on the importance of (13), note in $V_U^{kt} = V^{kt}/P_V^{kt}$ that V^{kt} is sectoral GDP in current prices, i.e., measured in the *same* "money units" (e.g., current US dollars) while P_V^{kt} is a sectoral GDP deflator that *differs* between sectors. Therefore, sectoral real GDP V_U^{kt} is not by itself in "homogeneous" units of measure across sectors. However, using the GDP deflator P_V^{Kt} as the common deflator in (13) converts *all* sectoral real GDP into "homogeneous" units of measure as V_U^{Kt}, i.e., real GDP is the *numeraire* good, and makes them "additive."[6] Thus, (13) resolves the non-additivity of GDP in chained prices.

In light of the above, relative price $r_V^{kt} \equiv P_V^{kt}/P_V^{Kt}$ may be interpreted as the price per unit of sectoral real GDP V_U^{kt} in terms of the economy's real GDP V_U^{Kt}. In principle, resources will tend to be reallocated from sectors with lower or falling relative prices to those with higher or rising relative prices. Thus, it appears logical to incorporate r_V^{kt} into the reallocation terms of BMB in (11), which is done in the reallocation terms of GEAD as shown below.

From (1) and (13), ALP in GEAD is given by,[7]

$$\frac{V_U^{Kt}}{L^{Kt}} = \sum_{k \in K} \left(\frac{P_V^{kt}}{P_V^{Kt}}\right)\left(\frac{L^{kt}}{L^{Kt}}\right)\left(\frac{V_U^{kt}}{L^{kt}}\right) \quad ; \quad t = (1,0). \tag{14}$$

GEAD arithmetic change in ALP from (14) is given by Eq. (2.7) or by Eq. (4.1) in Dumagan (2013). The latter may be rewritten using the notation in (14) as,

$$\frac{V_U^{K1}/L^{K1}}{V_U^{K0}/L^{K0}} - 1 = \sum_{k \in K} \frac{V^{k0}}{V^{K0}} \left(\frac{V_U^{k1}/L^{k1}}{V_U^{k0}/L^{k0}} - 1\right) + \sum_{k \in K} \frac{V_U^{k1}/L^{k1}}{V_U^{K0}/L^{K0}} \left(\frac{P_V^{k1}}{P_V^{K1}}\frac{L^{k1}}{L^{K1}} - \frac{P_V^{k0}}{P_V^{K0}}\frac{L^{k0}}{L^{K0}}\right). \tag{15}$$

In (15), reallocation effects in GEAD depend on changes in labor shares *and* in relative prices.

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[6] The choice of real GDP as the numeraire seems natural but not technically necessary. To define the relative price r_V^{kt} the denominator can be chosen arbitrarily, i.e., it need not be P_V^{Kt}.

[7] Except for differences in notation, (14) above is the same as Eq. (3) in Diewert (2010).

Since US GDP in chained prices is not additive (Ehemann, Katz, and Moulton, 2002; Balk, 2010), the Sato-Vartia US GDP quantity index will not be exact in the BMB framework. That is,

$$V_U^{Kt} \approx \sum_{k \in K} V_U^{kt} \quad ; \quad \ln\left(\frac{V_U^{K1}}{V_U^{K0}}\right) \approx \sum_{k \in K} \Psi^k \ln\left(\frac{V_U^{k1}}{V_U^{k0}}\right). \tag{16}$$

However, (13) permits incorporating relative prices into (16) to yield an exact GDP quantity index. Following Sato-Vartia, this index is,

$$V_U^{Kt} = \sum_{k \in K} \left(\frac{P_V^{kt}}{P_V^{Kt}}\right) V_U^{kt} \quad ; \quad \ln\left(\frac{V_U^{K1}}{V_U^{K0}}\right) = \sum_{k \in K} \Psi^k \ln\left[\frac{\left(P_V^{k1}/P_V^{K1}\right)V_U^{k1}}{\left(P_V^{k0}/P_V^{K0}\right)V_U^{k0}}\right]. \tag{17}$$

The weights Ψ^k remain the same because the share of $\left(P_V^{kt}/P_V^{Kt}\right)V_U^{kt}$ in V_U^{Kt} equals the nominal share of V^{kt} in V^{Kt}.

Hence, BMB in (11) becomes exact for GDP in chained or in constant prices when adjusted by changes in relative prices since (2), (13), and (17) yield this paper's GEA-BMB procedure,

$$\ln\left(\frac{V_U^{K1}/L^{K1}}{V_U^{K0}/L^{K0}}\right) = \sum_{k \in K} \Psi^k \ln\left(\frac{V_U^{k1}/L^{k1}}{V_U^{k0}/L^{k0}}\right) + \sum_{k \in K} \Psi^k \ln\left[\frac{\left(P_V^{k1}/P_V^{K1}\right)\left(L^{k1}/L^{K1}\right)}{\left(P_V^{k0}/P_V^{K0}\right)\left(L^{k0}/L^{K0}\right)}\right]. \tag{18}$$

Hence, for example, a rise in labor shares could be accompanied by a fall in relative prices such that the term inside square brackets in (18) is less than one resulting in negative reallocation effect that would have been positive in the absence of relative prices in (11). Thus, GEA-BMB shows that changes in relative prices could reverse the sign of reallocation effects in BMB.

Finally, notice that the corresponding productivity effects and reallocation effects of GEAD in (15) and GEA-BMB in (18) cannot differ in signs.

4. Comparing BMB, GEA-BMB, and GEAD Decompositions of Change in ALP

It is interesting to note that log-change in ALP cannot be larger than its arithmetic change. That is,[8]

$$\ln\left(\frac{V_U^{K1}/L^{K1}}{V_U^{K0}/L^{K0}}\right) \leq \frac{V_U^{K1}/L^{K1}}{V_U^{K0}/L^{K0}} - 1\,. \tag{19}$$

[8] Dumagan (2009) provides a mathematical proof that may be visualized as follows. In the x, y plane, let log-change be y_t^L and arithmetic change be y_t^A. Moreover, let $x_t = (V^{K1}/L^{K1})/(V^{K0}/L^{K0})$. Therefore, log-change is traced by the "curve," $y_t^L = \ln x_t$, while arithmetic change is traced by the "straight line," $y_t^A = x_t - 1$. The curve and straight line are tangent at $x_t = 1$ when both changes equal zero. This point is the intersection of the straight line and the x axis starting from a y intercept of -1. This straight line slopes upwards and for all $x_t \neq 1$, the curve lies everywhere below it and, thus, implies the inequality in (19).

There are two objectives in the following applications. One is to decompose the "actual" log-change in ALP in the left-hand side of (19) by BMB in (11) and by GEA-BMB in (18). The other is to decompose the "actual" arithmetic change in ALP in the right-hand side of (19) by GEAD in (15).

The data for 2004 and 2005 for US Manufacturing in Table A-1 were chosen to show that incorporating relative prices could switch the sign of reallocation effects and also the *sign* of a sector's total contribution to ALP change in Table A-2.

Table A-1. Manufacturing Sector Value Added and Employment in US, 2004-05

	Value Added in Current Prices (Billion Dollars)		**Value Added in CVM (Billion Chained 2005 Dollars)**		**Full-Time Equivalent (Thousand)**	
	2004	**2005**	**2004**	**2005**	**2004**	**2005**
Durable goods	822.0	878.3	816.8	878.3	8,919.0	8,958.0
Nondurable goods	660.6	691.0	701.0	691.0	5,385.0	5,277.0
Total	**1,482.6**	**1,569.3**	**1,516.8**	**1,569.3**	**14,304.0**	**14,235.0**

Source: Data on value added in current prices and in chained prices (CVM) and FTE employment are from the US Bureau of Economic Analysis. Note that value added in CVM is not additive. Thus, in CVM valued in chained 2005 dollars, the sum of the value added of durable goods and nondurable goods is not equal to the sector total value added in US manufacturing in 2004 and in 2005 above.

Table A-2. Decompositions of ALP Change in US Manufacturing, 2005

Without Relative Prices in BMB but With Relative Prices in GEA-BMB and in GEAD

	Productivity Effects (PE) (WSPGE)			**Reallocation Effects (RE) (DSRE + SSRE)**			**Total Contribution = PE + RE (WSPGE + DSRE + SSRE)**		
Equation number in text	**BMB** (11)	**GEA-BMB** (18)	**GEAD** (15)	**BMB** (11)	**GEA-BMB** (18)	**GEAD** (15)	**BMB** (11)	**GEA-BMB** (18)	**GEAD** (15)
Durable goods	3.7988	3.7988	3.9128	0.5124	-1.1096	-1.1706	4.3112	2.6892	2.7422
Nondurable goods	0.2596	0.2596	0.2619	-0.6832	0.9371	0.9583	-0.4236	1.1968	1.2203
Sum	**4.0585**	**4.0585**	**4.1748**	**-0.1708**	**-0.1725**	**-0.2123**	3.8877	3.8860	3.9625
Actual ALP change, 2005							**3.8860**	**3.8860**	**3.9625**
Difference from actual							**-0.0017**	**0.0000**	**0.0000**

Source: Author's calculations based on data in Table A-1 applied to procedures presented in this paper.

Note: WSPGE is *within-sector productivity growth effect*; DSRE is *dynamic structural reallocation effect*; and SSRE is *static structural reallocation effect*. These terms were adopted by Dumagan (2013) from an ADB study (Usui, 2011) and these terms correspond to *pure productivity growth effect*, *Baumol effect*, and *Denison effect* (Nordhaus, 2002). Dumagan (2013) computed DSRE and SSRE separately but these are combined above since this paper focuses on effects of relative prices on overall reallocation effects (RE).

Note that productivity effects (PE) are defined by the first terms while reallocation effects (RE) are defined by the second terms of BMB in (11), GEA-BMB in (18), and GEAD in (15).

The introduction of relative prices does not change PE from BMB to GEA-BMB as shown in Table A-2. However, RE could switch signs, for example, from positive (0.5124) of BMB to negative (−1.1096) of GEA-BMB for durable goods. Since BMB involves only labor shares, this means that relative prices fell by more than the rise in labor shares in durables. The switch from negative (−0.6832) to positive (0.9371) means that relative prices rose by more than the fall in labor shares in nondurable goods. The result is that the total contribution of durables remained positive but fell from 4.3112 to 2.6892 percentage points. However, total contribution of nondurables switched signs from −0.4236 to 1.1968 percentage points.

In US manufacturing in 2005, log-change in ALP was 3.8860 percent while arithmetic change was 3.9625 percent. It can be seen that BMB is inexact but GEA-BMB is an exact decomposition of log-change. Also, GEAD is an exact decomposition of arithmetic change. Thus, GEA-BMB and GEAD are exact for GDP in chained prices based on Fisher indexes.

The results above that incorporating relative prices could switch the sign of RE and could also switch the *sign* of a sector's total contribution to ALP (e.g., nondurables)–at the same time making the ALP change decomposition exact–underscores the importance of relative prices. That is, ignoring relative prices could yield inaccurate and misleading sectoral contributions.

The data in Table B-1 for the Agriculture sector in Italy and in Table C-1 also for the Agriculture sector in Thailand are the same data used in Dumagan (2013) to illustrate the exactness of GEAD for GDP in chained prices (Italy) and for GDP in constant prices (Thailand).

Table B-1. Agriculture Sector Value Added and Employment in Italy, 2008-09

	Value Added in Current Prices (Million Euros)		Value Added in CVM (Million Chained 2000 Euros)		Full-Time Equivalent (Thousand)	
	2008	**2009**	**2008**	**2009**	**2008**	**2009**
Agriculture, hunting and forestry	27,313.5	24,536.7	28,447.9	27,663.0	454.6	435.8
Fishing	1,203.6	1,349.0	759.3	817.8	33.4	34.7
Total	**28,517.1**	**25,885.6**	**29,052.0**	**28,378.7**	**488.0**	**470.5**

Source: Data on value added in current prices and in chained prices (CVM) and FTE employment are from Istat - Istituto Nazionale di Statistica. Note that value added in CVM is not additive. Thus, in CVM valued in chained 2000 euros, the sum of the value added of agriculture, hunting and forestry and fishing is not equal to the sector total value added in Italian agriculture in 2008 and in 2009 above.

Table C-1. Agriculture Sector GNP and Employment in Thailand, 2008-09

	GNP in Current Prices (Million Baht)		GNP in Constant Prices (Million 1988 Baht)		Employed Persons (Thousand)	
	2008	**2009**	**2008**	**2009**	**2008**	**2009**
Agriculture, hunting and forestry	955,710.0	931,907.0	320,058.0	322,342.0	14,283.3	14,228.3
Fishing	94,033.0	104,679.0	65,167.0	68,020.0	415.9	464.2
Total	**1,049,743.0**	**1,036,586.0**	**385,225.0**	**390,362.0**	**14,699.1**	**14,692.5**

Source: Data on GNP in current prices and in constant prices are from the National Economic and Social Development Board, Office of the Prime Minister. Data on employed persons are from the Report of the Labor Force Survey, National Statistical Office, Ministry of Information and Communication Technology.

The exact GEAD decompositions of arithmetic change in ALP in Italian Agriculture (Table B-2 from GDP in CVM in Table B-1) and in Thai Agriculture (Table C-2 from GDP in constant prices in Table C-1) are the same as those in Dumagan (2013).

The new results in Table B-2 and Table C-2 show that without relative prices, BMB is an inexact decomposition of log-change in ALP. Also new are the results that with relative prices, GEA-BMB is an exact decomposition of log-change in ALP for GDP in chained prices (Table B-2 from GDP in CVM in Table B-1 for Italy) and for GDP in constant prices (Table C-2 from GDP in constant prices in Table C-1 for Thailand).

Table B-2. Decompositions of ALP Change in Italian Agriculture, 2009

Without Relative Prices in BMB but With Relative Prices in GEA-BMB and in GEAD

	Productivity Effects (PE) (WSPGE)			**Reallocation Effects (RE) (DSRE + SSRE)**			**Total Contribution = PE + RE (WSPGE + DSRE + SSRE)**		
	BMB	**GEA-BMB**	**GEAD**	**BMB**	**GEA-BMB**	**GEAD**	**BMB**	**GEA-BMB**	**GEAD**
Equation number in text	(11)	(18)	(15)	(11)	(18)	(15)	(11)	(18)	(15)
Farms	1.3586	1.3586	1.3752	-0.5446	-1.1035	-1.1184	0.8140	0.2552	0.2568
Agriculture, hunting and forestry	0.1695	0.1695	0.1550	0.3511	0.8827	0.9042	0.5206	1.0522	1.0592
Fishing	**1.5281**	**1.5281**	**1.5303**	**-0.1936**	**-0.2207**	**-0.2143**	**1.3346**	**1.3074**	**1.3160**
Actual ALP change, 2009							**1.3074**	**1.3074**	**1.3160**
Difference from actual							**-0.0271**	**0.0000**	**0.0000**

Source: Author's calculations based on procedures presented in this paper applied to data in Table B-1, the same data used by Dumagan (2013).

Note: WSPGE is *within-sector productivity growth effect*; DSRE is *dynamic structural reallocation effect*; and SSRE is *static structural reallocation effect*. These terms were adopted by Dumagan (2013) from an ADB study (Usui, 2011) and these terms correspond to *pure productivity growth effect*, *Baumol effect*, and *Denison effect* (Nordhaus, 2002). Dumagan (2013) computed DSRE and SSRE separately but these are combined above since this paper focuses on effects of relative prices on overall reallocation effects (RE).

Table C-2. Decompositions of ALP Change in Thai Agriculture, 2009

Without Relative Prices in BMB but With Relative Prices in GEA-BMB and in GEAD

	Productivity Effects (PE) (WSPGE)			**Reallocation Effects (RE) (DSRE + SSRE)**			**Total Contribution = PE + RE (WSPGE + DSRE + SSRE)**		
	BMB	**GEA-BMB**	**GEAD**	**BMB**	**GEA-BMB**	**GEAD**	**BMB**	**GEA-BMB**	**GEAD**
Equation number in text	(11)	(18)	(15)	(11)	(18)	(15)	(11)	(18)	(15)
Agriculture, hunting and forestry	0.9919	0.9919	1.0035	-0.3080	-0.8937	-0.9046	0.6839	0.0982	0.0989
Fishing	-0.6387	-0.6387	-0.5813	1.0507	1.9099	1.8613	0.4121	1.2712	1.2800
Sum	**0.3532**	**0.3532**	**0.4222**	**0.7427**	**1.0162**	**0.9567**	**1.0959**	**1.3694**	**1.3788**
Actual ALP change, 2009							**1.3694**	**1.3694**	**1.3788**
Difference from actual							**0.2735**	**0.0000**	**0.0000**

Source: Author's calculations based on procedures presented in this paper applied to data in Table C-1, the same data used by Dumagan (2013).

Note: WSPGE is *within-sector productivity growth effect*; DSRE is *dynamic structural reallocation effect*; and SSRE is *static structural reallocation effect*. These terms were adopted by Dumagan (2013) from an ADB study (Usui, 2011) and these terms correspond to *pure productivity growth effect*, *Baumol effect*, and *Denison effect* (Nordhaus, 2002). Dumagan (2013) computed DSRE and SSRE separately but these are combined above since this paper focuses on effects of relative prices on overall reallocation effects (RE).

5. Conclusion: Implications on Earlier ALP Change Studies

The finding in this paper that incorporating relative prices could switch the sign of reallocation effects and could also switch the *sign* of a sector's total contribution to ALP change (e.g., nondurables in US manufacturing in 2005)–at the same time making ALP change decomposition exact–underscores the importance of relative prices. That is, ignoring differences and changes in relative prices could yield inaccurate and misleading sectoral contributions.

This paper showed that without relative prices, BMB is an inexact decomposition but with relative prices, GEA-BMB and GEAD are exact decompositions respectively of log-change and arithmetic change in ALP for GDP in chained or in constant prices. Moreover, the corresponding productivity effects and reallocation effects of GEA-BMB and GEAD cannot differ in signs, except for natural differences in size since log-change and arithmetic change may differ in size but not in sign.

This paper's equation (10) corresponds to Balk's Eq, (24) combined with Eq. (25) that, in turn, yields this paper's expression in (11). The latter was shown analytically in Section 2 and empirically in Section 4–by way of BMB in the Tables–to be inexact and problematic in the absence of relative prices when applied to GDP in chained or in constant prices.

However, Balk pointed out that Eq. (24) combined with Eq. (25) corresponds to Eq. (1) of Nordhaus (2002) and to Eq. (7) of Stiroh (2002). Furthermore, Balk's refinement of the combined equations to Eq. (31)–expressing value added labor productivity in terms of contributions of gross output and intermediate input–corresponds to Eq. (20) of Reinsdorf and Yuskavage (2010) and to Eq. (6) of Stiroh (2002) which was used by Bosworth and Triplett (2007), while Stiroh's Eq. (7) was used by Timmer, et.al. (2010). All equations corresponding to Balk's Eq. (24) with Eq. (25) or Balk's Eq. (31) are Törnqvist procedures except that of Reinsdorf and Yuskavage which is based on Sato-Vartia, although this one also has a corresponding Törnqvist procedure.

Therefore, earlier studies based on ALP change decompositions corresponding to Balk's Eq. (24) with Eq. (25) that ignore differences and changes in relative prices may need serious reconsideration, by implication of the analytical and empirical findings of this paper.

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